International Journal of Data Warehousing and Mining
Volume 20 • Issue 1

A Fuzzy Portfolio Model With Cardinality Constraints Based on Differential Evolution Algorithms

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ABSTRACT

Uncertain information in the securities market exhibits fuzziness. In this article, expected returns and liquidity are considered as trapezoidal fuzzy numbers. The possibility mean and mean absolute deviation of expected returns represent the returns and risks of securities assets, while the possibility mean of expected turnover represents the liquidity of securities assets. Taking into account practical constraints such as cardinality and transaction costs, this article establishes a fuzzy portfolio model with cardinality constraints and solves it using the differential evolution algorithm. Finally, using fuzzy c-means clustering algorithm, 12 stocks are selected as empirical samples to provide numerical calculation examples. At the same time, fuzzy c-means clustering algorithm is used to cluster the stock yield data and analyse the stock data comprehensively and accurately, which provides a reference for establishing an effective portfolio.

KEYWORDS

the efficient frontier of the fuzzy portfolio model, Fuzzy c-means clustering algorithm, liquidity, transaction costs, trapezoidal fuzzy numbers

INTRODUCTION

There are considerable data generated in the security markets, and many businesses rely on analysis of these data to excavate information. It is of great theoretical and practical significance that data mining technology is used to establish an effective portfolio investment model to identify the most valuable stock information, through which investors can make the best decision and effectively improve their return on investment (Wang, 2020; Kaur, 2022). The mean-variance portfolio model, initially introduced by Markowitz (Markowitz,1952), quantifies portfolios in terms of their means (returns) and variances (risks), serving as a foundational concept in quantitative investment research. Given the nature of complicated securities markets, investors often bring their subjective preferences into play. Historical returns and risks serve only as reference points for expected returns since they are subject to change and are inherently uncertain. Another influential factor in investment decisions is the liquidity of securities. Like expected returns, turnover rates are also subject to change and

DOI: 10.4018/IJDWM.341268 *Corresponding Author

inherently uncertain (Sui et al., 2020; Song et al., 2021). Konno and Yamazaki introduced the absolute deviation risk function, which overcomes the computational difficulties of the mean-variance model (Konno&Yamazaki,1991). Many scholars have explored portfolio selection using fuzzy theory. Carlsson and Fuller (Carlsson et al., 2002) treated returns as fuzzy numbers, defining possibility means and possibility variances, and proposed a fuzzy possibility portfolio selection model under a no-short-selling condition. Subsequently, various scholars have sought new methods for measuring expected returns (Zhang & Nie, 2003). Chen et al. (2007) and Zeng & Wang (2003) employed the possibility mean and possibility variance of asset returns to measure investment returns and risks, establishing portfolio selection models under financing conditions. In recent years, investors have sought portfolio solutions with a controlled number of securities, avoiding over-diversification when making portfolio selections.

This article utilizes trapezoidal fuzzy numbers to model expected returns and expected turnover rates, providing the possibility-based means of these parameters. We employ the absolute deviation risk function to construct an investment risk measure. To account for transaction costs in the securities market, we establish a fuzzy investment portfolio model with cardinal constraints. Finally, we demonstrate the practical application of this model in the context of the Chinese securities markets, underscoring its effectiveness and reliability. At the same time, the fuzzy c-means clustering algorithm can cluster the daily yield data of stocks, so the stock data samples can be analysed more accurately and comprehensively, which lays the foundation for constructing the portfolio (Begusic & Kostanjcar, 2019). Then, a more reasonable portfolio scheme is obtained.

PRELIMINARY KNOWLEDGE

In the fuzzy set theory, to describe the possibility of a fuzzy event occurring, Zedeh put forward the theory of possibility (Zadeh,1965), which is considered as a critical moment during which the fuzzy set theory experienced the development. Along with the development of the fuzzy set theory, a variety of phenomena of fuzzy uncertainties in the financial market is increasingly attracting the attention of a great number of scholars. Therefore, considerable studies conducted by these scholars employ the fuzzy set theory to address these phenomena of uncertainties that exist in the financial market. It is found that the fuzzy set theory is a powerful analytic tool in studying these phenomena of uncertainties in the stock. Today, the fuzzy portfolio has become a common research focus.

Assume that the fuzzy number A is a fuzzy set of the real numbers (denoted as R) with a bounded support membership function, and this membership function exhibits normality, fuzzy convexity, and continuity. The family of fuzzy numbers is defined as F. Let $A \in F$ be a fuzzy number, and $A(t)$ represents the membership function of A. Here, $\gamma \in [0,1]$ and $[A]^r = \{t \in R \mid A(t) \geq \gamma\}$ denotes a γ- level set of the fuzzy number A.

In the context of level sets of A, denoted as $A^\gamma = [a(\gamma), b(\gamma)]$, Carlsson and Fuller provide the following definitions for upper and lower possibility means in 2001:

$$M^U(A) = 2\int_0^1 \gamma b(\gamma)d\gamma = \frac{\int_0^1 Pos[A \geq b(\gamma)]b(\gamma)d\gamma}{\int_0^1 Pos[A \geq b(\gamma)]d\gamma}$$

$$M^L(A) = 2\int_0^1 \gamma a(\gamma)d\gamma = \frac{\int_0^1 Pos[A \geq a(\gamma)]a(\gamma)d\gamma}{\int_0^1 Pos[A \geq a(\gamma)]d\gamma}$$

Here, $Pos[A \geq b(\gamma)] = \prod((b(\gamma), +\infty)) = \sup_{u \geq b(\gamma)} A(u) = \gamma$

$$Pos[A \leq a(\gamma)] = \prod((-\infty, a(\gamma))) = \sup_{u \leq a(\gamma)} A(u) = \gamma$$

Definition 1: If $A \in F$, the probability mean of A is

$$\overline{M}(A) = \frac{M^L(A) + M^U(A)}{2} = \int_0^1 \gamma[a(\gamma) + b(\gamma)]d\gamma$$

Theorem 1: (1) Let A and B be two fuzzy numbers, then $\overline{M}(A+B) = \overline{M}(A) + \overline{M}(B)$. (2) Let A be a fuzzy number and k be a real number, then $\overline{M}(kA) = k\overline{M}(A)$.

POSSIBILITY MEAN BASED ON TRAPEZOIDAL FUZZY RETURNS

The trapezoidal fuzzy function rises at first for a period of time and then decreases, and its maximum membership degree can remain for a period of time so that it can fit more realistic scenarios. In the present study, the trapezoidal fuzzy number is used to describe the return rate of risk assets, aimed at helping readers better understand the proposed model in this chapter. Assuming there are n types of securities available for portfolio investment in the market, and the return rate for security asset i, denoted as $\tilde{r}_i = (a_i, b_i, a_i, \beta_i)(i = 1, 2, 3 \cdots n)$, is a trapezoidal fuzzy number with its membership function as the following:

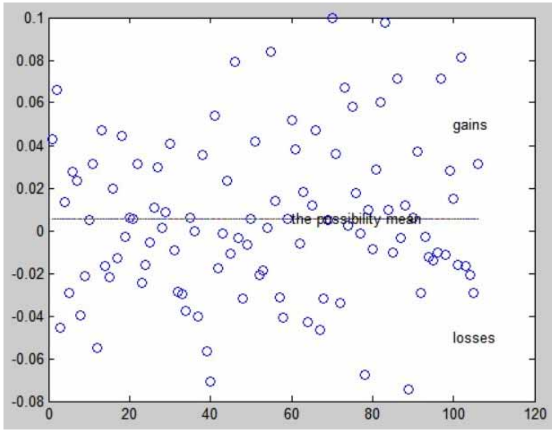
$$\tilde{r}_i(x) = \begin{cases} 1 - \dfrac{a_i - x}{\alpha_i}, a_i - \alpha_i \leq x < a_i \\ 1, a_i \leq x < b_i \\ 1 - \dfrac{x - b_i}{\beta_i}, b_i < x \leq b_i + \beta_i \\ 0, other \end{cases}$$

The λ sectional set of $\tilde{r}_i$ is $(\tilde{r}_i)_\lambda = [a_i - (1-\lambda)\alpha_i, b_i + (1-\lambda)\beta_i]$. According to the definition of the possibility mean, the possibility mean of the return rate $\tilde{r}_i$ of security asset i is:

$$\overline{M}(\tilde{r}_i) = \int_0^1 \gamma[a_i - (1-\lambda)\alpha_i + b_i + (1-\lambda)\beta_i]d\gamma$$
$$= \frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}$$

If x_i represents the investment ratio of an investor in securities assets, then the return on the portfolio investment $\tilde{R} = \sum_{i=1}^{n} x_i \tilde{r}_i$ is also a trapezoidal fuzzy number.

Figure 1. Mean absolute deviation



It is known that the possibility mean of $\tilde{R}$ is:

$$\overline{M}(\tilde{R}) = \overline{M}(\sum_{i=1}^{n} x_i \tilde{r}_i) = \sum_{i=1}^{n} x_i (\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}) \,. \tag{1}$$

ABSOLUTE DEVIATION BASED ON TRAPEZOIDAL FUZZY RETURNS

Although it is effective to use variance to measure risks of the portfolio in the security and financial markets, variance measurement focuses on the degree of deviation between a random variable and its mathematical expectation. This characteristic is not applicable to the issue of portfolio selections. In real-world situations, investors are less fond of extremely low profits, but fully expect extremely high profits. Therefore, variance used to measure risks of the portfolio is not accurate, but it is more scientific that average absolute deviation ratio variance is used to measure risks. Using the mean absolute deviation as depicted in Figure 1 to measure risk is more scientifically reasonable than variance (Chen et al., 2012). The mean absolute deviation is defined as:

$$w(x) = \overline{M}(\left|\min\{0, \sum_{i=1}^{n} (\tilde{r}_i - \overline{M}(\tilde{r}_i))x_i\}\right|) = \overline{M}(\max\{0, \sum_{i=1}^{n} (\overline{M}(\tilde{r}_i) - \tilde{r}_i)x_i\})$$

$\sum_{i=1}^{n} (\overline{M}(\tilde{r}_i) - \tilde{r}_i)x_i$ is still a trapezoidal fuzzy number, and

$$\sum_{i=1}^{n} (\overline{M}(\tilde{r}_i) - \tilde{r}_i)x_i = [\sum_{i=1}^{n} (\overline{M}(\tilde{r}_i) - b_i)x_i, \sum_{i=1}^{n} (\overline{M}(\tilde{r}_i) - a_i)x_i, \sum_{i=1}^{n} x_i\beta_i, \sum_{i=1}^{n} x_i\alpha_i]$$

Based on He (2009), the following conclusions can be drawn:

$$\max\{0, \sum_{i=1}^{n} (\overline{M}(\tilde{r}_i) - \tilde{r}_i)x_i\} = [0, \sum_{i=1}^{n} (\overline{M}(\tilde{r}_i) - a_i)x_i, 0, \sum_{i=1}^{n} x_i\alpha_i]$$

The mean absolute deviation:

$$w(x) = \overline{M}(\max\{0, \sum_{i=1}^{n} \overline{M}(x_i \tilde{r}_i) - x_i \tilde{r}_i\})$$
$$= \frac{\sum_{i=1}^{n}(\overline{M}(\tilde{r}_i) - a_i)x_i}{2} + \frac{\sum_{i=1}^{n} x_i \alpha_i}{6} = \frac{\sum_{i=1}^{n} x_i (b_i - a_i)}{4} + \frac{\sum_{i=1}^{n} x_i (\alpha_i + \beta_i)}{12} \quad (2)$$

CONSIDERING TRANSACTION COSTS

In the process of assets trading, transaction fees and collection methods affect investors' asset allocation, which further affects the yield. Traditionally, the portfolio model normally takes the balance between profits and risks into consideration, without calculating the trade cost. Consequently, investors lose a great amount of principal, and meanwhile, suffer the reduction of the interest rate. As per this issue, the importance of the trade cost is demonstrated and taking the trade cost into consideration is applicable to the actual situation of the stock market. State tax authorities, exchanges, and security companies collect transaction fees from investors, in the form of stamp duties, transfer fees, brokerage trading commissions, and the like. (1) Stamp duty: investors only need to pay one-thousandth of the transaction value when they sell the shares, (2) Transfer fee: this fee is only paid when the investor conducts Shanghai stock and fund transactions and is charged at 1/10,000 of the transaction amount. If it is less than CNY 1, it is charged at CNY 1, (3) Brokerage trading commission: security management fees and security transaction handling fees are also included. The maximum is not more than three thousandths of the transaction amount, the minimum is from CNY 5, and the commission of a single transaction less than CNY 5 is charged at CNY 5. Although transaction fees are small in number, ignoring transaction costs often results in ineffective securities portfolios (Sim et al., 2023).

In the security market, various entities such as the national tax authorities, stock exchanges, and securities firms charge investors transaction fees in proportion to certain factors, such as trading commissions, stamp duties, and transfer fees. In fact, transaction costs, though small in quantity, are often overlooked and can lead to ineffective securities portfolios. Assuming that the transaction costs for security k_i represent a fixed proportion of the transaction amount, C_i is the transaction cost required for security i. The initial investment portfolio of the investor is $(x_1^0, x_2^0 \cdots x_n^0)$. The transaction cost for the new investment portfolio of $(x_1, x_2 \cdots x_n)$ is $C_i = k_i \left| x_i - x_i^0 \right|$. The total transaction cost is $\sum_{i=1}^{n} C_i = \sum_{i}^{n} k_i \left| x_i - x_i^0 \right|$.

The possibility mean of $\tilde{R}$ is improved to be:

$$\overline{M}(\tilde{R}) = \overline{M}(\sum_{i=1}^{n} x_i \tilde{r}_i) = \sum_{i=1}^{n} x_i (\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}) - \sum_{i=1}^{n} k_i \mid x_i - x_i^0 \mid \quad (3)$$

LIQUIDITY RISK

In the portfolio, apart from revenues and risks that are primarily concerned by investors, the liquidity of securities cannot be ignored (Sui et al., 2020). The liquidity of securities refers to converting securities into cash, that is, the amount of securities that investors can sell quickly under the condition of guaranteed profit or small loss. In the process of investment, it is hoped that yields are gained from the asset as large as possible, but at the same time, the liquidity of assets should be taken into consideration. The turnover rate refers to the ratio of turnover and circulation of assets, which can reflect the liquidity of assets. It is generally acknowledged that the higher the turnover rate, the more frequent transactions take place, so funds enter and exit the market more easily. In other words, the liquidity of assets is better. On the contrary, if the turnover rate is lower and there are fewer transactions, the liquidity of the asset is worse. In addition, uncertainties of asset liquidity are susceptible to the effects of investors' subjective considerations regarding the degree of the liquidity. If the liquidity of assets is uncertain, it can be said that the liquidity of assets is regarded as a vague number. Liquidity, defined as the ease of converting securities into cash, is a crucial metric for assessing the quality of securities in the market. Incorporating liquidity constraints into models can help investors construct more robust portfolios, mitigating liquidity risk. Turnover rate is commonly used to measure liquidity, where higher turnover indicates more frequent trading, easier capital flow in and out of the market, and better liquidity, while lower turnover signifies poor liquidity. To use turnover rate to measure the liquidity of securities, as the size of securities' liquidity is often subjective, it is treated as a fuzzy phenomenon. Assuming the fuzzy turnover rate of security asset i is a trapezoidal fuzzy number: $\tilde{l}_i = (la_i, lb_i, l\alpha_i, l\beta_i)$, the possibility mean of the fuzzy turnover rate of security asset i is:

$$M(\tilde{l}_i) = \frac{la_i + lb_i}{2} + \frac{l\beta_i - l\alpha_i}{6}$$

Then the probability mean of the investment portfolio is:

$$M(\sum_{i=1}^{n} x_i \tilde{l}_i) = \sum_{i=1}^{n} x_i (\frac{la_i + lb_i}{2} + \frac{l\beta_i - l\alpha_i}{6}) \tag{4}$$

CONSTRUCTING A DECISION MODEL

If the company is not doing well, concentrating on buying its securities assets may result in devastating failure, so investors cannot only buy one type of stock in specific operations. At the same time, because buying and selling securities assets requires certain transaction costs, such as stamp duties and handling fees, investors cannot hold too many types of securities assets at the same time. Based on investment experience, smart investors often choose to hold around 30% of the candidate stocks, which can not only achieve high returns but also reduce risks, giving a base constraint K to the total number of securities assets. Assuming that the minimum value of investment weight x_i is 0, which means short selling is not allowed, and u_i is the maximum value of x_i, then the threshold constraint of the investment portfolio is $0 \le x_i \le u_i$.

The idea of the model is to select the optimal investment ratio to minimize the total risk under the constraints of predetermined lower expected returns and expected turnover rates. Combining (1), (2), (3), and (4) decision models can represent:

$$\min \quad w(x) = \frac{\sum_{i=1}^{n} z_i x_i (b_i - a_i)}{4} + \frac{\sum_{i=1}^{n} z_i x_i (\alpha_i + \beta_i)}{12}$$

$$s.t \begin{cases} \sum_{i=1}^{n} z_i x_i (\frac{a_i + b_i}{2} + \frac{\beta_i - \alpha_i}{6}) - \sum_{i=1}^{n} z_i k_i \mid x_i - x_i^0 \mid \geq r_0 \\ \sum_{i=1}^{n} z_i x_i (\frac{la_i + lb_i}{2} + \frac{l\beta_i - l\alpha_i}{6}) \geq l_0 \\ \sum_{i=1}^{n} z_i x_i = 1 \\ \sum_{i=1}^{n} z_i = K \\ l_i \leq x_i \leq u_i \\ z_i \in \{0,1\}, i = 1,2 \cdots n, K \in Z^+ \end{cases}$$

DIFFERENTIAL EVOLUTION ALGORITHM

The differential evolution algorithm was proposed by Storn in 1997 to solve the Chebyshev polynomial problem (hereinafter referred to as DE algorithm) (Raktim et al., 2023; Lulu et al., 2023). The population evolution of the differential evolution algorithm is realized by the cooperation and competition of individuals in the population. The algorithm is robust in global searching ability, because the algorithm has a unique memory ability and can self-adjust the search direction. Due to its simple algorithm structure, few control parameters, good convergence performance, and global searching ability, the differential evolution algorithm has been applied to various areas including bio-information, power, signal processing and mechanical design. Additionally, along with the expansion of applicable areas, it is found that the algorithm is applicable to solve some complex multi-objective optimization problems.

The DE algorithm not only has global search ability and strong robustness, but it is also not limited by the attributes of the problem. Therefore, whether the problem is simple or of complex optimization, the DE algorithm is applicable to both. The basic philosophy of the algorithm is Darwin's "survival of the fittest" biological evolution. To be specific, the method is to change, cross, and select each parent in the population to produce new offspring. Then the newly generated child is regarded as the parent, and the mutation, crossover, and selection operations are performed again until the optimal solution is found. Through this way of continuous replacement of the population, the inferior individuals are eliminated, and the superior individuals are retained. Finally, the global optimal solution is obtained. The steps of the differential evolution algorithm are shown in Figure 2 (Gupta et al., 2023):

Step 1: Initialization

Determine the population size N,

$(X_i^t, Z_i^t) = (x_{1i}^t, x_{2i}^t, \cdots, x_{ni}^t, z_{1i}^t, z_{2i}^t, \cdots, z_{ni}^t)(i = 1 \cdots n)$, t represents the t^{th} generation. (X_{best}^t, Z_{best}^t) represents the optimal individual in the t^{th} generation. The initial population is generated as follows:

$$X_i^0 = l_i + rand() \times (u_i - l_i)$$
$$Z_i^0 = [rand()],$$

In the equation, $rand()$ $[0,1]$ represents a uniformly distributed random number between 0 and 1, and $[*]$ denotes the operation of rounding to the nearest integer.

Step 2: Mutation Operation

Mutation operation is:

$$(X_v^t, Z_v^t) = \lambda(X_{r3}^t, Z_{r3}^t) + (1-\lambda)(X_{best}^t, Z_{best}^t) + F[(X_{r1}^t, Z_{r1}^t) - (X_{r2}^t, Z_{r2}^t)]$$

Here, $\lambda = \dfrac{T_{\max} - t}{T_{\max}}$, $T_{\max}$ is the maximum number of iterations. During the search process, the weight of (X_{r3}^t, Z_{r3}^t) gradually decreases while the weight of (X_{best}^t, Z_{best}^t) gradually increases, changing from 1 to 0, ensuring that the algorithm has both strong global search and fast convergence rate and search accuracy.

Variable mutation operation of real numbers is:

$$X_v^t = \lambda X_{r3}^t + (1-\lambda)X_{best}^t + F(X_{r1}^t - X_{r2}^t).$$

01 Variable mutation operation

$$Z_v^t = [\lambda Z_{r3}^t + (1-\lambda)Z_{best}^t + F(Z_{r1}^t - Z_{r2}^t)].$$

Step 3: Crossover Operation

DE (differential evolution) utilizes cross operation to maintain population diversity. For the i^{th} individual (X_i^t, Z_i^t) in the group, cross operate it with (X_v^t, Z_v^t) to generate the experimental individual (X_C^t, Z_C^t).

The cross operation is:

$$x_{jC}^t = \begin{cases} x_{jv}^t, rand() \le CR \\ x_{ji}^t, othersize \end{cases}, j = 1 \cdots n,$$

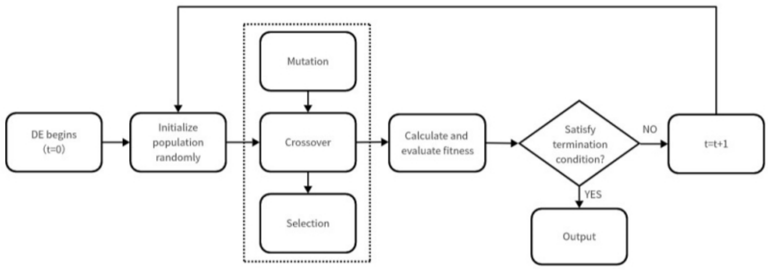
$$Z_{jC}^t = \begin{cases} Z_{jv}^t, rand() \le CR \\ Z_{ji}^t, othersize \end{cases}, j = 1 \cdots n$$

CR is determined by the following equation:

$$CR = CR_{\min} + (CR_{\max} - CR_{\min})e^{-a(t - \frac{t}{T_{\max}})^b}, a = 30, b = 3.$$

In this way, in the initial stage, (X_i^t, Z_i^t) contributes more to (X_C^t, Z_C^t), improving global search ability, while in the later stage, (X_i^t, Z_i^t) contributes more to (X_C^t, Z_C^t), improving local search ability. Set $CR_{\max}$ as the maximum crossover probability and $CR_{\min}$ as the minimum crossover probability.

Figure 2. Differential evolution algorithm flowchart



Step 4: Selection Operation

The fitness function is the opposite number of the objective function of model (2), and the equation for selecting the operation is:

$$(X_i^{t+1}, Z_i^{t+1}) = \begin{cases} (X_C^t, Z_C^t), Fitness(X_C^t, Z_C^t) > Fitness((X_i^t, Z_i^t)) \\ X_i^t, Z_i^t, othersize \end{cases}$$

Step 5: Termination Test

If the accuracy requirements are met or the entire evolution has reached the evolution deadline, stop the machine and output (X_i^{t+1}, Z_i^{t+1}) as an approximation solution, otherwise, set $t = t + 1$ and turn to step 2.

COMPUTATIONAL INSTANCES

Fuzzy c-means (Xiang, 2022; Xingyu et al., 2020; Chen, et al., 2021), abbreviated as FCM algorithm, is regarded as a generalized form of the k-means clustering algorithm. K-means is a traditional hard clustering algorithm with the membership values of only 0 and 1, and the basic criterion of its division is to minimize the sum of squared errors within the class. However, the FCM clustering algorithm is a soft clustering algorithm with its membership value of any value between 0 and 1, and the basic criterion of this algorithm is to minimize the sum of squares of the weighted errors in the class. To demonstrate the versatility of the proposed model, a comprehensive analysis is conducted, taking into account factors such as stock return indicators, turnover rate indicators, and industry characteristics. Twelve stocks are selected as empirical samples from the Shanghai Stock Exchange using FCM, and weekly return rate indicators for these securities from June 2019 to June 2021, as well as turnover rate indicators, are collected. Fuzzy return rate indicators and fuzzy liquidity indicators of assets are characterized using trapezoidal fuzzy numbers.

The fuzzy return rate estimation method proposed by Vercher et al. (2007) and others is employed to determine the trapezoidal fuzzy numbers for stock return rate indicators and liquidity indicators. The computed results are presented in Tables 1 and 2 (Zhou, 2022; El Kharrim, 2023).

As regards to $k_i = 0.003$, $x_i^0 = 0$, $u_i = 0.3$, $l_i = 0$, $K = 4$, the investment ratios and risks are calculated using Matlab software for various expected return levels, as shown in Table 3.

From Table 2, it can be observed that the efficient frontier of the fuzzy portfolio model with cardinality constraints is depicted in Figure 3.

From Table 2 and Figure 1, it can be observed that as expected returns increase, the portfolio's risk correspondingly rises. Different investors may choose different investment portfolios based on

Table 1. Fuzzy returns of 12 stocks

Transaction Code	a	b	α	β
600036	-0.0044	0.00248	0.00646	0.04953
603535	-0.00393	0.00483	0.01326	0.08128
603693	-0.00497	0.00431	0.01323	0.09518
600763	-0.00163	0.00878	0.01350	0.05115
600988	-0.00709	0.00742	0.01633	0.08407
600316	-0.0051	0.00709	0.01740	0.08999
600845	-0.00419	0.00833	0.01571	0.05621
603939	-0.00461	0.00665	0.01282	0.05242
603127	-0.00157	0.01071	0.01852	0.06324
600359	-0.0077	0.00531	0.01720	0.09482
600185	-0.00509	0.00818	0.01816	0.088
603185	-0.00718	0.00459	0.01247	0.09612

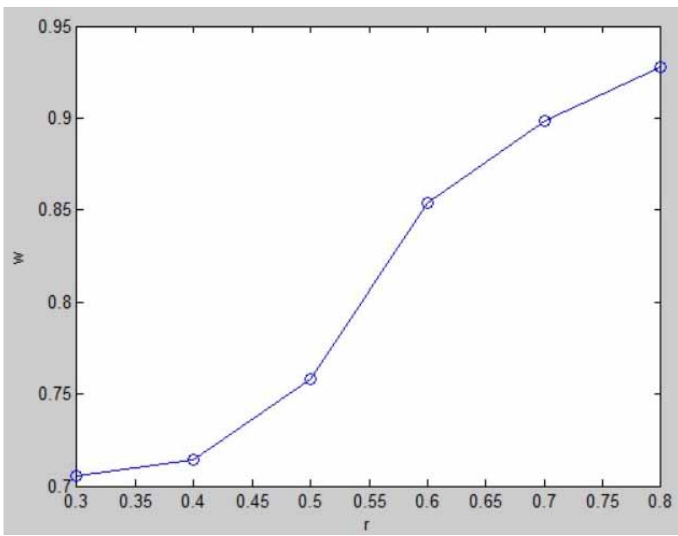
Table 2. Fuzzy turnover rates of 13 stocks

Transaction Code	la	lb	$l\alpha$	$l\beta$
600036	0.00232	0.00296	0.00104	0.00334
603535	0.01437	0.01912	0.00865	0.04283
603693	0.01543	0.03228	0.00948	0.09835
600763	0.00831	0.0098	0.00289	0.00966
600988	0.03746	0.05455	0.02197	0.0654
600316	0.0209	0.02959	0.01273	0.04103
600845	0.00663	0.00826	0.00303	0.00952
603939	0.00544	0.00687	0.00278	0.00553
603127	0.0152	0.02436	0.00862	0.02695
600359	0.03354	0.05395	0.02269	0.11965
600185	0.00463	0.01198	0.0035	0.06063
603185	0.05472	0.07367	0.03411	0.07904

Table 3. Investment ratios and risks (Unit: %)

Expected Returns	Investment Ratios	Risks
0.3	x1=60, x2=0, x3=0, x4=30, x5=0, x6=0, x7=0, x8=10, X9=0, x10=0, x11=0, x12=0	0.7053667
0.4	x1=60, x2=0, x3=0, x4=30, x5=0, x6=0, x7=10, x8=0, X9=0, x10=0, x11=0, x12=0	0.7140833
0.5	x1=60, x2=0, x3=0, x4=0, x5=0, x6=0, x7=0, x8=12.27, x9=27.72, x10=10, x11=0, x12=0	0.7584611
0.6	x1=30, x2=0, x3=0, x4=30, x5=0, x6=0, x7=0, x8=0, X9=30, x10=10, x11=0, x12=0	0.8536500
0.7	x1=26.63, x2=0, x3= 13.37, x4=30, x5=0, x6=0, x7=0, x8=0, X9=30, x10=0, x11=0, x12=0	0.8983754
0.8	x1=12.56, x2=0, x3=27.44, x4=30, x5=0, x6=0, x7=0, x8=0, X9=30, x10=0, x11=0, x12=0	0.9279422

Figure 3. Efficient Frontier of the Fuzzy Portfolio Model With Cardinality Constraints



their individual risk preferences. However, the performance of the Chinese stock market has been poor in recent years, with the majority of stock returns falling below 0.8. Nevertheless, investors are still required to bear significant risks.

CONCLUSION

This paper treats returns and liquidity as fuzzy numbers. Under certain expected returns and expected turnover rate conditions, with constraints imposed by the cardinality and investment ratio, and with the objective of minimizing risk, a mixed nonlinear programming investment selection model is established. Based on this, FCM was used to select a portfolio of 12 stocks in the Chinese stock market, demonstrating the practical application of this model and illustrating its effectiveness and reliability, offering a new approach for securities investment decision-making.

The innovations of the study are as follows:

(1) The return rate is taken as the trapezoidal fuzzy number, which not only comprehensively analyses the risk faced by investors, but also simplifies the model calculation.
(2) Its liquidity is taken as a trapezoidal fuzzy number and the turnover rate is used to measure the liquidity of securities, which reflects the flow of funds in and out of the market.
(3) The differential evolution algorithm is used to effectively solve the hybrid nonlinear programming investment selection model.
(4) The objective fuzzy frequency statistical approximation of the study is used to estimate trapezoidal fuzzy number and relevant parameters of fuzzy variables under fixed probability distribution, which is scientific.

Further aspects of the research can be studied as follows:

(1) In the real investment environment, in addition to the basis constraints and threshold constraints, there are real restrictions such as whole lot trading, budget constraints, and short selling. Therefore, portfolio selection with realistic constraints under the framework of fuzzy goals is a problem worth further study.
(2) In the financial market, although there are many fuzzy uncertain phenomena, it is undeniable that there exist random phenomena. Therefore, fuzzy random variables or random fuzzy variables can be used to draw uncertainty factors in financial markets. Then, a multi-objective portfolio selection model with stochastic and fuzzy uncertainty can be established. The future research direction is to use fuzzy stochastic programming theory to solve the model.
(3) The convergence of a differential evolution algorithm is used to solve portfolio problems with cardinality constraints.
(4) More algorithms are used to solve portfolio problems with cardinality constraints.

REFERENCES

Begusic, S., & Kostanjcar, Z. (2019). Cluster-based shrinkage of correlation matrices for portfolio optimization. *International Symposium on Image and Signal Processing and Analysis,* (pp. 301-305). IEEE. doi:10.1109/ISPA.2019.8868482

Carlsson, C., Fuller, R., & Mailender, P. (2002). A possibilistic approach to selecting portfolios with highest utility score. *Fuzzy Sets and Systems*, *131*(1), 13–21. doi:10.1016/S0165-0114(01)00251-2

Chen, J. S., Zhang, H., Pi, D. C., Kantardzic, M., Yin, Q., & Liu, X. (2021). A weight possibilistic fuzzy c-means clustering algorithm. *Scientific Programming*, *2021*, 1–10. doi:10.1155/2021/9965813

Chen, Y. J., Liu, Y. K., & Wu, X. L. (2012). A new risk criterion in fuzzy environment and its application. *Applied Mathematical Modelling*, *36*(7), 3001–3022. doi:10.1016/j.apm.2011.09.081

El Kharrim, M. (2023). Multi-period fuzzy portfolio optimization model subject to real constraints. *EURO Journal on Decision Processes*, *11*, 1–9. doi:10.1016/j.ejdp.2023.100041

Jasleen, K., & Hushdeep, D. (2022). Application and performance of data mining techniques in stock market: A review. *International Journal of Intelligent Systems in Accounting Finance & Management*, *29*(4), 219–241. doi:10.1002/isaf.1518

Jianhua, Z., & Shouyang, W. (2003). A portfolio model based on fuzzy decision theory. *Systems Engineering Theory and Practice*, *1*, 99–104.

Konno, H., & Yamazaki, H. (1991). Mean absolute portfolio optimization model and its application to Yoko stock market. *Management Seienee*, *37*(5), 519–513.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, *1952*(7), 77–91.

Punit, G., Furqan, R., Khadija, K., Wajdi, A., Sultan, A., Mejd, S., & Imran, A. (2024). Detecting thyroid disease using optimized machine learning model based on differential evolution. *International Journal of Computational Intelligence Systems*, *1*, 1–19.

Raktim, B., & Deepak, S. (2023). Chaos control assisted single-loop multi-objective reliability-based design optimization using differential evolution. *Swarm and Evolutionary Computation*, *8*, 101340.

Sim, H. S., Ling, W. S. Y., Leong, W. J., & Chen, C. Y. (2023). Proximal linearized method for sparse equity portfolio optimization with minimum transaction cost. *Journal of Inequalities and Applications*, *1*(1), 1–16. doi:10.1186/s13660-023-03055-4

Song, H., Long, X., He, G., & Peng, Z. (2021). A fuzzy portfolio model with background risk and liquidity considered. *Applied Mathematics and Mechanics*, *42*(2), 212–220.

Song, L. L., Meng, Y., Guo, Q. X., & Gong, X. C. (2023). Improved differential evolution algorithm for slab allocation and hot-rolling scheduling integration problem. *Mathematics*, *11*(9), 2050. doi:10.3390/math11092050

Sui, Y., Hu, J., & Ma, F. (2020). A possibilistic portfolio model with fuzzy liquidity constraint. *Complexity*, *2020*, 1–10. doi:10.1155/2020/3703017

Vercher, E., Bermudez, J. D., & Segura, J. V. (2007). Fuzzy portfolio optimization under downside risk measures. *Fuzzy Sets and Systems*, *158*(7), 769–782. doi:10.1016/j.fss.2006.10.026

Wang, D. X. (2020). Quantitative investment model based on data mining. *Revista Internacional de Metodos Numericos para Calculo y Diseno en Ingenieria*, *36*(1), 1–7. doi:10.23967/j.rimni.2020.03.006

Wei, C., Runtong, Z., & Ling, Y. (2007). A fuzzy decision-making method for securities portfolio selection under financing conditions. *Journal of Beijing Jiaotong University*, *1*, 67–70.

Weiguo, Z., & Zankan, N. (2003). Model for selecting asset portfolios and analytical representation of optimal solutions. *Journal of Operations Research*, *7*(3), 91–101.

Xiang, Y. (2022). *Fuzzy c-means clustering method for stock portfolio optimization*. Dalian University of Technology.

Xingyu, A., Weilong, L., Sidou, C., & Yong, Z. (2020). A multi-period fuzzy mean-minimax risk portfolio model with investor's risk attitude. *Soft Computing*, *25*, 2949–2963.

Yingru, Z. (2022). *A credibility portfolio selection model considering liquidity constraints*. Nanjing University of Technology.

Yingyu, H. (2009). The portfolio strategy problem with transaction costs in a fuzzy environment. *Mathematical Statistics and Management*, *28*(3), 538–543.

Zadeh, L. A. (1965). Fuzzy Sets. *Information and Control*, *8*(3), 338–353. doi:10.1016/S0019-9958(65)90241-X