A New Fuzzy Joint Choquet Integral Method Under Interval-Valued Function

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ABSTRACT

A new fuzzy group decision-making method considering multi-attributes correlation under interval-valued function is presented, which mainly includes (1) acquiring the group fuzzy preference matrix and (2) handling the interactions between multiple evaluation attributes. To do that, firstly, the fuzzy joint Choquet integral based on an interval-valued function is proposed, which not only reflects the interaction between multiple attributes in a complex and uncertain environment, but also retains the initial preference of the decision maker. Secondly, a Shapley value with fuzzy measure is applied to assign each decision maker's weight, and the fuzzy group preference matrix is acquired by fusing the fuzzy preference matrices of all decision makers. Finally, a nursing home selection case is depicted to explain the effectiveness of the proposed technique. The corresponding sensitivity analysis is operated, which clarifies the reliability and flexibility of the proposed technique.

KEYWORDS

Choquet Integral, Group Decision Making, Interval-Valued Function, Shapley Value

1. INTRODUCTION

The multi-attribute group decision making (MAGDM) is to fuse preferences depicted through some decision makers about evaluation attributes for alternatives (Li, 2007; Fu et al. 2020). The problem of MAGDM can be seen everywhere in daily life (Tong et al.,2022; Zheng er al.,2023; Chao et al.,2021). For example, people are concerned about the choice of nursing homes, the choice of cruise ships (Cao et al.,2022) or the choice of sustainable suppliers (Xu et al.,2019). In the traditional MAGDM, the evaluation attributes belong to a mutually independent state. Due to the advent of social media, the environment of MAGDM becomes complicated, which makes the evaluation attributes have a certain degree of uncertainty, resulting in a certain relationship between evaluation attributes. Therefore, how to build a new MAGDM method with multi-evaluation attributes correlation is the key problem addressed in this paper.

One of the key issues is to deal with the correlation between multiple evaluation attributes (Ju et al.,2020; Chen et al. 2020; Li et al., 2020). In the traditional process of MAGDM, the weights of evaluation attributes are subjectively given or objectively obtained. This indicates that these evaluation attributes are independent of each other. However, Due to the advent of social media, the environment

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of MAGDM becomes more and more complex and uncertain, which makes the evaluation attributes have some correlation with each other (Marichal & Roubens, 2000; She et al.,2021; Teng& Liu, 2021). For example, the evaluation attributes in nursing homes include living environment, hardware facilities and the literacy of accompanying personnel. The living environment can reflect the good attitude of the caregivers to work, and the high-quality caregivers also show that their serious attitude in work makes the living environment of the elderly comfortable. This indicates that there is a positive relationship between the evaluation attributes of nursing homes.

Choquet integral is an effective way to solve the interaction between multiple evaluation attributes (Teng& Liu, 2021; Wang et al.,2018; Byüközkan et al.,2021]. For example, Teng et al. (2021) designed a generalized Choquet integral to explore the correlation between evaluation attributes in MAGDM. At the same time, the Pythagorean fuzzy number based Choquet integral is proposed to discuss the interaction between multiple evaluation attributes (Byüközkan et al.,2021). These Choquet integral methods can not preserve the initial preferences of each decision maker in the merging of all preference fusion, while fuzzy numbers with interval valued functions can deal with this problem well (Dubois & Prade, 1980; Qiu,2021; Qiu &Yu, 2023; Li et al.,2018; 2020; Fei et al.,2018; Nan,2014). In the traditional Choquet integral, the multiple evaluation attributes are local interaction, only some attributes coalitions are considered. In order to consider all attribute coalitions, hence, the fuzzy joint Choquet integral based on an interval-valued function is put forward to deal with the correlation between multiple evaluation attributes in this paper.

Another key issue is how to reflect the global interaction relationship between decision makers. In traditional MAGDM stage, decision makers' weights are also subjectively given or objectively calculated (Hendiani et al.,2020; Liu et al.2019; Xu et al.2019). This also shows that the decision makers belong to an independent state. With the emergence of Internet media, there is a certain trust relationship between decision makers, and some scholars regard this trust relationship as a reliability resource to dominate the weight of decision makers (Lu et al., 2021; Zhang et al., 2020, Teng et al., 2022; Xu et al.,2020; Liu et al.,2019). For instance, the trust relationship is used to induce the weight of decision makers (Liu et al.,2019). Ma et al. (2021) formalized the trust relationship as an interaction weight between two decision makers. Although this reflects the interaction relationship of all decision makers in the merging of MAGDM. However, this kind of interaction is a local relationship, which does not make decision makers interact globally to ensure the fairness of weight allocation. Therefore, the Shapley value is applied in this paper to fairly dominate decision makers' weights.

Based on the above literature analysis and inspiration, a new fuzzy group decision making method considering multi-evaluation attributes correlation under interval-valued function is put forward, which it mainly includes two innovations:

(1) The fuzzy joint Choquet integral based on an interval-valued function is put forward. First, fuzzy Choquet integral ($F-CI_{\mu}$) and the fuzzy inverse Choquet integral ($FR-CI_{\mu}$) under an interval-valued function are put forward, respectively. And then the fuzzy joint Choquet integral ($FJ-CI_{\mu}$) under an interval-valued function is proposed, which is composed of the linear convex combination of $F-CI_{\mu}$ and $FR-CI_{\mu}$. This $FJ-CI_{\mu}$ not only reflects the global interaction between multiple attributes, but also retains the initial preference of the decision maker.
(2) The new fuzzy group decision making method is presented. First, the Shapley value with fuzzy measure is applied to assign the weight of each decision maker, which illustrates the global interaction among decision makers to ensure the fairness of each decision maker. And then, the fuzzy group preference matrix is acquired by fusing the fuzzy preference matrices of all decision makers.

The framework for the article is structured as follows. Some related concepts for fuzzy numbers and Choquet integral are depicted in Section 2. Section 3 puts forward the fuzzy joint Choquet integral under interval-valued function. A new $FJ-CI_{\mu}$ based MAGDM approach under interval-valued function is proposed in Section 4. A nursing home selection case is operated in Section 5. Conclusions and future works are depicted in Section 6.

2. PRELIMINARIES

Some concepts and comments with respect to fuzzy numbers and Choquet integral are shown below.

2.1. Fuzzy Numbers

The set B_c is depicted as the class of all bounded closed intervals in $\mathbb{R}$ (Diamond & Kloeden, 1990), namely,

$$B_c = \left\{ \left[p_L, p_R \right] \mid p_L, p_R \in \mathbb{R} \wedge p_L \leq p_R \right\}.$$

A fuzzy set $\tilde{\phi}$ of $\mathbb{R}$ is denoted as a membership function $\vartheta_{\tilde{\phi}} : \mathbb{R} \to \left[0,1\right]$, and the η-level set of $\tilde{\phi}$ is depicted through $\left[\tilde{\phi}\right]^{\eta} = \left\{ \phi \in \mathbb{R} : \vartheta_{\tilde{\phi}}\left(\phi\right) \geq \eta \right\}$ for $\forall \eta \in \left(0,1\right]$. The $\left[\tilde{\phi}\right]^{0} = \overline{\bigcup_{\eta \in \left(0,1\right]} \left[\tilde{\phi}\right]^{\eta}}$ is presented through the closure of a $\bigcup_{\eta \in \left(0,1\right]} \left[\tilde{\phi}\right]^{\eta}$. A fuzzy number $\tilde{\phi}$ is a fuzzy set with nonempty sets $\left[\tilde{\phi}\right]^{\eta} = \left[\phi_L\left(\eta\right), \phi_R\left(\eta\right)\right] \forall \eta \in \left[0,1\right]$, where it consists of a left endpoint function $\phi_L\left(\eta\right)$ and a right endpoint function $\phi_R\left(\eta\right)$ (Dubois and Prade, 1980) And the class of all $\tilde{\phi}$ is depicted as F.

Definition 1. (Wang & Wu, 2003). Let $\tilde{\phi}, \tilde{\varphi} \in F$. Then $\tilde{\phi}$ is inferior to $\tilde{\varphi}$ ($\tilde{\phi} \preceq \tilde{\varphi}$) if and only if $\left[\tilde{\phi}\right]^{\eta} = \left[\phi_L\left(\eta\right), \phi_R\left(\eta\right)\right] \leq \left[\tilde{\varphi}\right]^{\eta} = \left[\varphi_L\left(\eta\right), \varphi_R\left(\eta\right)\right] \Leftrightarrow \phi_L\left(\eta\right) \leq \varphi_L\left(\eta\right)$ and $\phi_R\left(\eta\right) \leq \varphi_R\left(\eta\right)$ for each $\eta \in \left[0,1\right]$.

Which the $\preceq$ is reflexive and transitive. And $\forall \tilde{\phi}, \tilde{\varphi} \in F$ are comparable under the $\preceq$ (Syau & Stanley, 2006a; 2006b).

Definition 2. (Zadeh, 1965). Let $\tilde{\phi} \in F$, If its the membership function, $\vartheta_{\tilde{\phi}}\left(\phi\right)$, is

$$\vartheta_{\tilde{\phi}}\left(\phi\right) \begin{cases} 0, & \phi < \underset{.}{o}, \phi > \dot{o}, \\ \dfrac{\phi - \underset{.}{o}}{o - \underset{.}{o}}, & \underset{.}{o} \leq \phi \leq o, \\ \dfrac{\dot{o} - \phi}{\dot{o} - o}, & o < \phi \leq \dot{o}. \end{cases}$$

Thus, $\tilde{\phi}$ is called a triangular fuzzy number. And $\tilde{\phi}$ is depicted as $\tilde{\phi} = \left(\underset{.}{o}, o, \dot{o}\right)$.

Corresponding to its η-level set is

$$\left[\tilde{\phi}\right]^{\eta} = \left[\phi_L\left(\eta\right), \phi_R\left(\eta\right)\right] = \left[\left(o - \underset{.}{o}\right)\eta + \underset{.}{o}, -\left(\dot{o} - o\right)\eta + \dot{o}\right], \eta \in \left(0,1\right]. \tag{1}$$

And the image of η-level set of $\tilde{\phi} = \left(\underset{\cdot}{o}, o, \dot{o}\right)$ is shown in Figure 1.

Definition 3. (Li et al., 2015). Let $\tilde{\phi}_i \in F$ and $x_i \in \mathbb{R}\left(x_i \geq 0, i = 1,2,\cdots,n\right)$. Then, it has $\sum_{i=1}^{n} \tilde{\phi}_i x_i \in F$.

2.2. Choquet Integral

Choquet integral (Choquet, 1953) is an effective way to solve the interaction between multiple evaluation attributes in MAGDM. The relevant explanation is as follows.

Definition 4. (Choquet, 1953) If a set $Q = \left\{q_1, q_2, \cdots, q_n\right\}$ is given, it assumes that $P\left(Q\right)$ denotes the set of all sub-coalitions on Q. Hence, the fuzzy measure (FM) μ is defined through function $\mu : P\left(Q\right) \rightarrow \left[0,1\right]$, and it yields

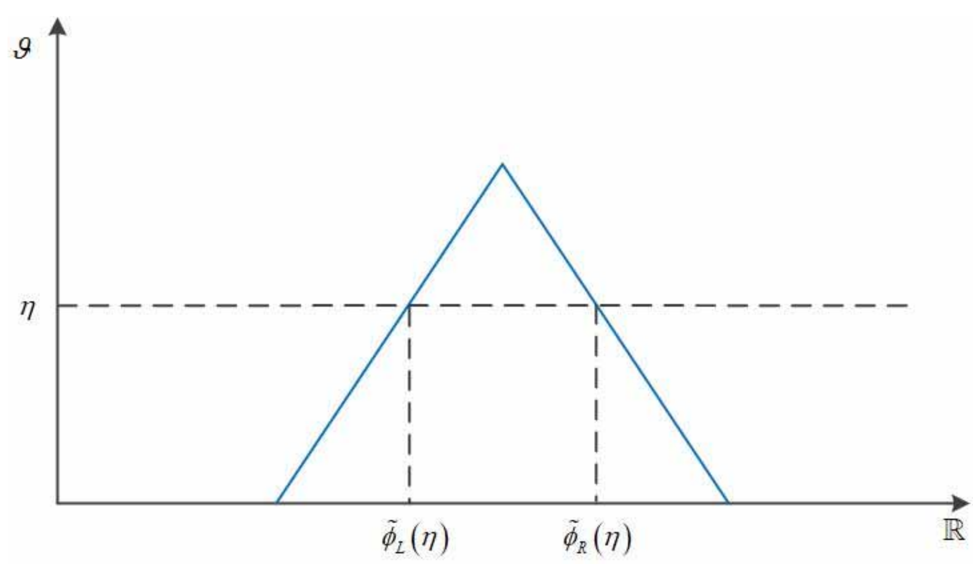
(1) $\mu\left(\varnothing\right) = 0$;
(2) $\mu\left(Q\right) = 1$;
(3) $\forall \alpha, \beta \in P\left(Q\right), \alpha \subseteq \beta \Rightarrow \mu\left(\alpha\right) \leq \mu\left(\beta\right)$;
(4) $\forall \alpha, \beta \in P\left(Q\right), \mu\left(\alpha \cup \beta\right) = \mu\left(\alpha\right) + \mu\left(\beta\right) + \varepsilon\mu\left(\alpha\right)\mu\left(\beta\right), \varepsilon \in \left[-1, \infty\right)$ and $\alpha \cap \beta = \varnothing$.

Then, the ε-FM μ is:

$$\mu\left(Q\right) = \frac{1}{\varepsilon}\left[\prod_{j=1}^{n}\left(1 + \varepsilon\mu\left(q_j\right)\right) - 1\right], \quad if \varepsilon \neq 0. \tag{2}$$

Particularly, $\mu\left(Q\right) = \sum_{j=1}^{n} \mu\left(q_j\right), \varepsilon = 0$. Thus, ε is acquired with the $\mu\left(Q\right) = 1$, that is,

Figure 1. The image of η-level set of $\tilde{\phi} = \left(\underset{\cdot}{o}, o, \dot{o}\right)$



$$\varepsilon + 1 = \prod_{j=1}^{n} \left(1 + \varepsilon \mu\left(q_j\right)\right). \tag{3}$$

Definition 5. (Choquet, 1953) Assume that $\Theta\left(\Theta \geq 0\right)$ is a real valued function and μ is a FM on Q. Then, the Choquet integral (CI_μ) of $\Theta\left(q_1\right), \Theta\left(q_2\right), \cdots, \Theta\left(q_n\right)$ is:

$$CI_\mu\left(\Theta\left(q_1\right), \Theta\left(q_2\right), \cdots, \Theta\left(q_n\right)\right) = \sum_{j=1}^{n} \left(\mu\left(\varpi_{\sigma(j)}\right) - \mu\left(\varpi_{\sigma(j+1)}\right)\right)\Theta\left(q_{\sigma(j)}\right) \tag{4}$$

where $\left\{\sigma\left(1\right), \sigma\left(2\right), \cdots, \sigma\left(n\right)\right\}$ is a permutation of $\left\{1, 2, \cdots, n\right\}$, meanwhile, it yields $\Theta\left(q_{\sigma(1)}\right) \leq \Theta\left(q_{\sigma(2)}\right) \leq \cdots \leq \Theta\left(q_{\sigma(n)}\right), \varpi_{\sigma(j)} = \left\{q_{\sigma(j)}, q_{\sigma(j+1)}, \cdots, q_{\sigma(n)}\right\}$ and $\varpi_{\sigma(n+1)} = \varnothing$.

3. THE FUZZY JOINT CHOQUET INTEGRAL UNDER INTERVAL-VALUED FUNCTION

The interval-valued function can well describe preference of each decision maker, and has the advantage for retaining the original preference of decision makers when all preferences are aggregated. Choquet integral can well depict the interaction among multiple evaluation attributes. In the traditional Choquet integral, the multiple evaluation attributes are local interaction, only some attributes coalitions are considered. In order to consider all attribute coalitions, Meng et al. (2021) proposed the bidirectional Choquet integral under discrete numbers. Inspired by this, this paper proposes fuzzy joint Choquet integral. First, the fuzzy Choquet integral based on interval-valued functions are proposed as follows.

3.1. Fuzzy Choquet Integral Based on Interval-Valued Functions

Definition 6. ($F - CI_\mu$) Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j, \tilde{\phi}_j \succeq \tilde{0}\ \left(j = 1, 2, \cdots, n\right)$ are fuzzy numbers on Q. Then, the fuzzy Choquet integral $F - CI_\mu$ of $\tilde{\phi}_1, \tilde{\phi}_2, \cdots$ and $\tilde{\phi}_n$ is:

$$F - CI_\mu\left(\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_n\right) = \oplus_{j=1}^{n} \left[\mu\left(\varpi_{\sigma(j)}\right) - \mu\left(\varpi_{\sigma(j+1)}\right)\right]\tilde{\phi}_{\sigma(j)} \tag{5}$$

where $\left\{\sigma\left(1\right), \sigma\left(2\right), \cdots, \sigma\left(n\right)\right\}$ is a permutation of $\left\{1, 2, \cdots, n\right\}$, meanwhile, it yields $\tilde{\phi}_{\sigma(1)} \leq \tilde{\phi}_{\sigma(2)} \leq \cdots \leq \tilde{\phi}_{\sigma(n)}, \varpi_{\sigma(j)} = \left\{q_{\sigma(j)}, q_{\sigma(j+1)}, \cdots, q_{\sigma(n)}\right\}$ and $\varpi_{\sigma(n+1)} = \varnothing$.

By Definition 1, Definition 3 and Eq. (4), the proposed $F - CI_\mu$ yields some detailed Theorems, which are shown below.

Theorem 1. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j = 1, 2, \cdots, n\right)$ are fuzzy numbers on Q. Thus, it gets $F - CI_\mu\left(\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_n\right) \in F$.

Proof. By Definition 1, Definition 3 and Eq (4), it gets $F - CI_\mu\left(\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_n\right) \in F$.

Theorem 2. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j = 1, 2, \cdots, n\right)$ are fuzzy numbers on Q. If $\tilde{\phi}_j = \tilde{\phi}\left(j = 1, 2, \cdots, n\right)$. Hence, $F - CI_\mu\left(\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_n\right) = \tilde{\phi}$.

Proof. By Definition 1, Definition 3 and Eq. (4), it obtains

$$\begin{aligned}F-CI_{\mu}\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right) = F-CI_{\mu}\left(\phi_1,\tilde{\phi},\cdots,\tilde{\phi}\right)&\\ &= \oplus_{j=1}^{n}\left[\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right]\tilde{\phi}\\ &= \left\{\sum_{j=1}^{n}\left[\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right]\right\}\tilde{\phi}\\ &= \tilde{\phi}.\end{aligned}$$

Theorem 3. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. If $\tilde{\phi}_1{}', \tilde{\phi}_2{}', \cdots$ and $\tilde{\phi}_n{}'$ are any permutation of $\tilde{\phi}_1, \tilde{\phi}_2, \cdots$ and $\tilde{\phi}_n$. Hence, $F-CI_{\mu}\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right) = F-CI_{\mu}\left(\tilde{\phi}_1{}',\tilde{\phi}_2{}',\cdots,\tilde{\phi}_n{}'\right)$.

Proof. Since $\tilde{\phi}_1{}', \tilde{\phi}_2{}', \cdots$ and $\tilde{\phi}_n{}'$ are any permutation of $\tilde{\phi}_1, \tilde{\phi}_2, \cdots$ and $\tilde{\phi}_n$, by Definition 1, Definition 3 and Eq. (4), it obtains $F-CI_{\mu}\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right) = F-CI_{\mu}\left(\tilde{\phi}_1{}',\tilde{\phi}_2{}',\cdots,\tilde{\phi}_n{}'\right)$.

Theorem 4. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j, \tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. Hence, $\tilde{\phi}_{\sigma(1)} \preceq F-CI_{\mu}\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right) \preceq \tilde{\phi}_{\sigma(n)}$.

Proof. By Definition of $F-CI_{\mu}$, it has $\tilde{\phi}_{\sigma(1)} \preceq \tilde{\phi}_j \preceq \tilde{\phi}_{\sigma(n)}$, and based on Definition 1, it has

$$\left(\phi_{\sigma(1)}\right)_L\left(\eta\right) \preceq \left(\phi_j\right)_L\left(\eta\right) \preceq \left(\phi_{\sigma(n)}\right)_L\left(\eta\right)$$

and

$$\left(\phi_{\sigma(1)}\right)_R\left(\eta\right) \preceq \left(\phi_j\right)_R\left(\eta\right) \preceq \left(\phi_{\sigma(n)}\right)_R\left(\eta\right).$$

Moreover, $\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right) > 0$, it gets

$$\begin{aligned}\left(\phi_{\sigma(1)}\right)_L\left(\eta\right) &= \sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_{\sigma(1)}\right)_L\left(\eta\right)\\ &\le \sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_{\sigma(j)}\right)_L\left(\eta\right)\\ &\le \sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_{\sigma(n)}\right)_L\left(\eta\right)\\ &= \left(\phi_{\sigma(n)}\right)_L\left(\eta\right).\end{aligned}$$

$$\begin{aligned}\left(\phi_{\sigma(1)}\right)_R\left(\eta\right) &= \sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_{\sigma(1)}\right)_R\left(\eta\right)\\ &\le \sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_{\sigma(j)}\right)_R\left(\eta\right)\\ &\le \sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_{\sigma(n)}\right)_R\left(\eta\right)\\ &= \left(\phi_{\sigma(n)}\right)_R\left(\eta\right).\end{aligned}$$

By Definition 1, it has $\tilde{\phi}_{\sigma(1)} \preceq F-CI_{\mu}\left(\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_n\right) \preceq \tilde{\phi}_{\sigma(n)}$.

Theorem 5. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j, \tilde{\varphi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. If $\tilde{\phi}_j \preceq \tilde{\varphi}_j$, hence, $F-CI_{\mu}\left(\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_n\right) \preceq F-CI_{\mu}\left(\tilde{\varphi}_1, \tilde{\varphi}_2, \cdots, \tilde{\varphi}_n\right)$.

Proof. Since $\tilde{\phi}_j \preceq \tilde{\varphi}_j$ and by Definition 1, it gets

$\left(\phi_j\right)_L\left(\eta\right) \leq \left(\varphi_j\right)_L\left(\eta\right)$ and $\left(\phi_j\right)_R\left(\eta\right) \leq \left(\varphi_j\right)_R\left(\eta\right)$.

Moreover, $\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)>0$, it gets

$$\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_j\right)_L\left(\eta\right) \leq \left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\varphi_j\right)_L\left(\eta\right)$$
$$\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_j\right)_R\left(\eta\right) \leq \left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\varphi_j\right)_R\left(\eta\right).$$

Thus, it gets

$$\sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_j\right)_L\left(\eta\right) \leq \sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\varphi_j\right)_L\left(\eta\right)$$
$$\sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\phi_j\right)_R\left(\eta\right) \leq \sum_{j=1}^{n}\left(\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right)\left(\varphi_j\right)_R\left(\eta\right).$$

By Definition 1, it has $F-CI_{\mu}\left(\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_n\right) \preceq F-CI_{\mu}\left(\tilde{\varphi}_1, \tilde{\varphi}_2, \cdots, \tilde{\varphi}_n\right)$.

3.2. Fuzzy Reverse Choquet Integral Based on Interval Valued Function

Next, the fuzzy reverse Choquet integral based on interval valued function is defined.

Definition 7. ($FR-CI_{\mu}$) Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}$ $\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. Then, the fuzzy reverse Choquet integral $FR-CI_{\mu}$ of $\tilde{\phi}_1, \tilde{\phi}_2, \cdots$ and $\tilde{\phi}_n$ is:

$$FR-CI_{\mu}\left(\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_n\right) = \oplus_{j=1}^{n}\left[\mu\left(W_{\sigma(j)}\right)-\mu\left(W_{\sigma(j-1)}\right)\right]\tilde{\phi}_{\sigma(j)} \tag{6}$$

where $\left\{\sigma\left(1\right), \sigma\left(2\right), \cdots, \sigma\left(n\right)\right\}$ is a permutation of $\left\{1,2,\cdots,n\right\}$, meanwhile, it yields $\tilde{\phi}_{\sigma(1)} \leq \tilde{\phi}_{\sigma(2)} \leq \cdots \leq \tilde{\phi}_{\sigma(n)}, W_{\sigma(j)} = \left\{q_{\sigma(1)}, q_{\sigma(2)}, \cdots, q_{\sigma(j)}\right\}$ and $W_{\sigma(0)} = \varnothing$.

By Definition 1, the proposed $FR-CI_{\mu}$ also yields some detailed Theorems, which are shown below.

Theorem 6. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. Thus, it gets $FR-CI_{\mu}\left(\tilde{\phi}_1, \tilde{\phi}_2, \cdots, \tilde{\phi}_n\right) \in F$.

Proof. Similar to the proof of Theorem 1, hence, it is omitted.

Theorem 7. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. If $\tilde{\phi}_j = \tilde{\phi}\left(j=1,2,\cdots,n\right)$. Hence, $FR-CI_\mu\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right)=\tilde{\phi}$.

Proof. Similar to the proof of Theorem 2, hence, it is omitted.

Theorem 8. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. If $\tilde{\phi}_1{}', \tilde{\phi}_2{}', \cdots$ and $\tilde{\phi}_n{}'$ are any permutation of $\tilde{\phi}_1, \tilde{\phi}_2, \cdots$ and $\tilde{\phi}_n$. Hence, $FR-CI_\mu\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right)=FR-CI_\mu\left(\tilde{\phi}_1{}',\tilde{\phi}_2{}',\cdots,\tilde{\phi}_n{}'\right)$.

Proof. Similar to the proof of Theorem 3, hence, it is omitted.

Theorem 9. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. Hence, $\tilde{\phi}_{\sigma(1)} \preceq FR-CI_\mu\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right) \preceq \tilde{\phi}_{\sigma(n)}$.

Proof. Similar to the proof of Theorem 4, hence, it is omitted.

Theorem 10. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j, \tilde{\varphi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. If $\tilde{\phi}_j \preceq \tilde{\varphi}_j$, hence, $FR-CI_\mu\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right) \preceq FR-CI_\mu\left(\tilde{\varphi}_1,\tilde{\varphi}_2,\cdots,\tilde{\varphi}_n\right)$.

Proof. Similar to the proof of Theorem 5, hence, it is omitted.

Based on the Definitions of $F-CI_\mu$ and $FR-CI_\mu$, the fuzzy joint Choquet integral based on interval valued function is proposed as follows.

3.3. Fuzzy Joint Choquet Integral Based on Interval Valued Function

Definition 8. ($FJ-CI_\mu$) Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\ \left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. Then, the fuzzy joint Choquet integral $FJ-CI_\mu$ of $\tilde{\phi}_1, \tilde{\phi}_2, \cdots$ and $\tilde{\phi}_n$ is:

$$FJ-CI_\mu\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right)=\lambda\left\{\oplus_{j=1}^n\left[\mu\left(\varpi_{\sigma(j)}\right)-\mu\left(\varpi_{\sigma(j+1)}\right)\right]\tilde{\phi}_{\sigma(j)}\right\}\oplus\left(1-\lambda\right)\left\{\oplus_{j=1}^n\left[\mu\left(W_{\sigma(j)}\right)-\mu\left(W_{\sigma(j-1)}\right)\right]\tilde{\phi}_{\sigma(j)}\right\} \tag{7}$$

where $\left\{\sigma\left(1\right),\sigma\left(2\right),\cdots,\sigma\left(n\right)\right\}$ is a permutation of $\left\{1,2,\cdots,n\right\}$, meanwhile, it yields $\tilde{\phi}_{\sigma(1)} \leq \tilde{\phi}_{\sigma(2)} \leq \cdots \leq \tilde{\phi}_{\sigma(n)}, \varpi_{\sigma(j)}=\left\{q_{\sigma(j)},q_{\sigma(j+1)},\cdots,q_{\sigma(n)}\right\}, W_{\sigma(j)}=\left\{q_{\sigma(1)},q_{\sigma(2)},\cdots,q_{\sigma(j)}\right\}, \varpi_{\sigma(n+1)}=\varnothing$ and $W_{\sigma(0)}=\varnothing$.

Note 1: The above $FJ-CI_\mu$ based on an interval-valued function is composed of the linear convex combination of $F-CI_\mu$ and $FR-CI_\mu$. And the frame diagram of corresponding new $FJ-CI_\mu$ aggregation operator is shown in Figure 2.

By Definition 1, the proposed $FJ-CI_\mu$ also yields some detailed Theorems, which are shown below.

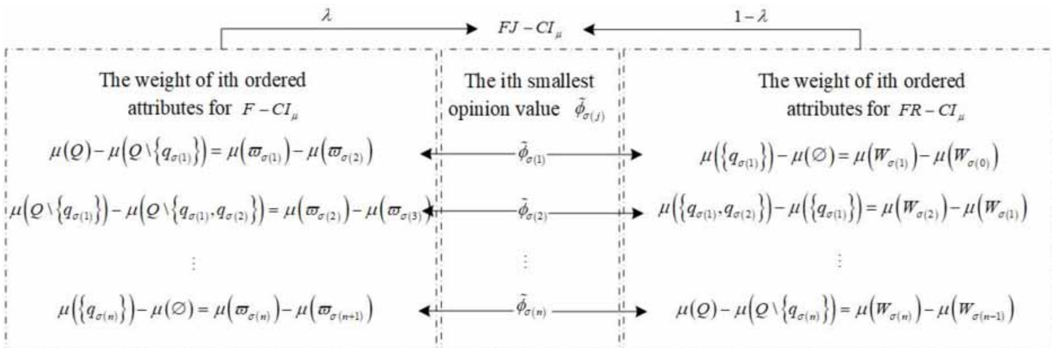
Theorem 11. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. Thus, it gets $FJ-CI_\mu\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right)\in F$.

Proof. Similar to the proof of Theorem 1, hence, it is omitted.

Theorem 12. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. If $\tilde{\phi}_j = \tilde{\phi}\left(j=1,2,\cdots,n\right)$. Hence, $FJ-CI_\mu\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right)=\tilde{\phi}$.

Proof. Similar to the proof of Theorem 2, hence, it is omitted.

Figure 2. The frame diagram of new $FJ-CI_{\mu}$ under interval-valued function



Theorem 13. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. If $\tilde{\phi}_1{}',\tilde{\phi}_2{}',\cdots$ and $\tilde{\phi}_n{}'$ are any permutation of $\tilde{\phi}_1,\tilde{\phi}_2,\cdots$ and $\tilde{\phi}_n$. Hence, $FJ-CI_{\mu}\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right)=FJ-CI_{\mu}\left(\tilde{\phi}_1{}',\tilde{\phi}_2{}',\cdots,\tilde{\phi}_n{}'\right)$.

Proof. Similar to the proof of Theorem 3, hence, it is omitted.

Theorem 14. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. Hence, $\tilde{\phi}_{\sigma(1)} \preceq FJ-CI_{\mu}\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right) \preceq \tilde{\phi}_{\sigma(n)}$.

Proof. Similar to the proof of Theorem 4, hence, it is omitted.

Theorem 15. Given a set Q, and it assumes that μ is FM and $\tilde{\phi}_j,\tilde{\varphi}_j \succeq \tilde{0}\left(j=1,2,\cdots,n\right)$ are fuzzy numbers on Q. If $\tilde{\phi}_j \preceq \tilde{\varphi}_j$, hence, $FJ-CI_{\mu}\left(\tilde{\phi}_1,\tilde{\phi}_2,\cdots,\tilde{\phi}_n\right) \preceq FJ-CI_{\mu}\left(\tilde{\varphi}_1,\tilde{\varphi}_2,\cdots,\tilde{\varphi}_n\right)$.

Proof. Similar to the proof of Theorem 5, hence, it is omitted.

4. A NEW $FJ-CI_{\mu}$ -BASED MAGDM APPROACH UNDER INTERVAL-VALUED FUNCTION

A new $FJ-CI_{\mu}$ based MAGDM approach under interval-valued function is designed in this study, which mainly includes (1) Obtain the group fuzzy preference matrix Stage; and (2) Alternatives selection Stage. And its a flow chart is presented in Figure 3. The specific implementation stage is as follows:

Stage 1: Obtain the group fuzzy preference matrix

Stage 1.1: Identify the set of alternatives and criteria

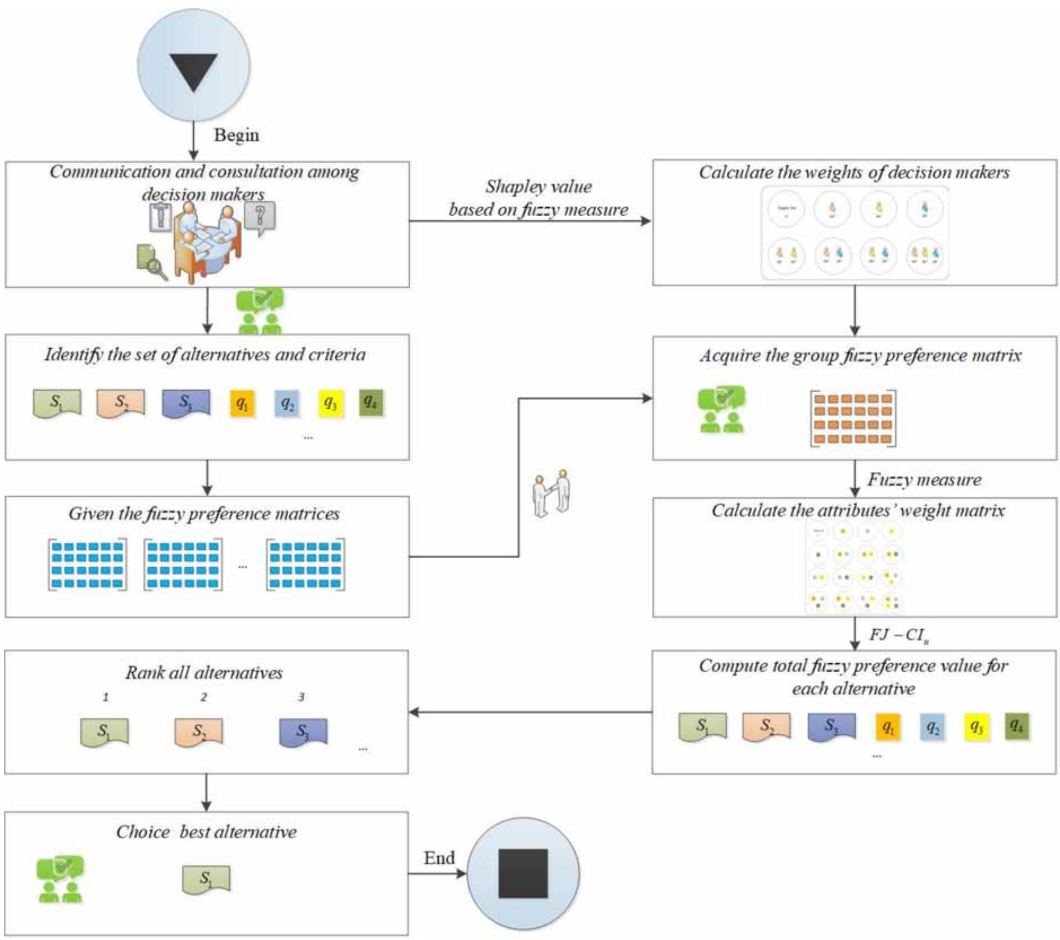
Some decision makers form groups $D=\left\{dm^1,dm^2,\cdots,dm^h\right\}$ that communicate with each other based on their background knowledge and references reading to identify a set of alternatives $S=\left\{S_1,S_2,\cdots,S_m\right\}$ and criteria $Q=\left\{q_1,q_2,\cdots,q_n\right\}$.

Stage 1.2: Given the fuzzy preference matrix $\tilde{F}^l$ of dm^l

Based on the definition of fuzzy number with interval-valued function, decision maker $dm^l\left(l=1,2,\cdots,h\right)$ from different fields, ages and education levels express their fuzzy preference matrices $\tilde{F}^l\left(l=1,2,\cdots,h\right)$ for $\left(S_i,q_j\right)\left(i=1,2,\cdots,m;j=1,2,\cdots,n\right)$.

Stage 1.3: Acquire the group fuzzy preference matrix

Figure 3. A flow chart of new $FJ-CI_{\mu}$-based MAGDM approach under interval-valued function



Suppose that $\tilde{F}^{l}=\left(\tilde{\phi}_{ij}\right)_{m\times n}\left(l=1,2,\cdots,h\right)$ are fuzzy preference matrices given by $dm^{l}\left(l=1,2,\cdots,h\right)$, respectively. Since Shapley value can well reflect the global interaction relationship between decision makers, moreover, Shapley value with FMs is utilized to calculate the weights of $dm^{l}\left(l=1,2,\cdots,h\right)$ (Shapley, 1953):

$$\chi_{l}=\sum_{E\subseteq D\setminus dm^{l}}\frac{\#E!\left(h-\#E-1\right)!}{h!}\left(\mu\left(E\cup dm^{l}\right)-\mu\left(E\right)\right) \tag{8}$$

which $\#E$ and h present the number in the decision maker's coalition E and D. And χ_{l} represents the weight of the dm^{l} in a cooperative game, which indicates that there is some fairness among the decision makers in the interaction process. And $\sum_{l=1}^{h}\chi_{l}=1$. Hence, the decision makers' weights vector is $\chi=\left(\chi_{1},\chi_{2},\cdots,\chi_{h}\right)$. Further, by Definition 3, the fuzzy preference matrices

$\tilde{F}^l = \left(\tilde{\phi}_{ij}^l\right)_{m\times n} \left(l = 1,2,\cdots,h\right)$ of $dm^l \left(l = 1,2,\cdots,h\right)$ are aggregated into a group fuzzy preference matrix $\tilde{F}^c = \left(\tilde{\phi}_{ij}^c\right)_{m\times n}$:

$$\begin{aligned}&\tilde{\phi}_{ij}^c = \chi_1\tilde{\phi}_{ij}^1 \oplus \chi_2\tilde{\phi}_{ij}^2 \oplus \cdots \oplus \chi_h\tilde{\phi}_{ij}^h \\ &\left[\tilde{\phi}_{ij}^c\right]^\eta = \left[\sum_{l=1}^{h}\chi_l\left(\left(\phi_{ij}\right)_L\left(\eta\right)\right)^l, \sum_{l=1}^{h}\chi_l\left(\left(\phi_{ij}\right)_R\left(\eta\right)\right)^l\right].\end{aligned} \tag{9}$$

Stage 2: Calculate the attributes' weight matrix

By the definition of FM, the FM of the attributes' coalition is calculated. Moreover, the each order attribute's weight is computed through the difference between the FM*s* of the adjacent order attributes' coalitions associated with this attribute. For a $\tilde{F}^c = \left(\tilde{\phi}_{ij}^c\right)_{m\times n}$, group fuzzy preference values are ranked from small to large about for each $q_j \left(j = 1,2,\cdots,n\right)$ about $S_i \left(i = 1,2,\cdots,m\right)$. Hence, the attributes' weight matrix is:

$$\zeta = \begin{pmatrix} \zeta_{11} & \cdots & \zeta_{1n} \\ \vdots & \ddots & \vdots \\ \zeta_{m1} & \cdots & \zeta_{mn} \end{pmatrix} \tag{10}$$

and the attributes' reverse weight matrix is:

$$\psi = \begin{pmatrix} \psi_{11} & \cdots & \psi_{1n} \\ \vdots & \ddots & \vdots \\ \psi_{m1} & \cdots & \psi_{mn} \end{pmatrix}. \tag{11}$$

Stage 3: Compute total fuzzy preference value of S_i.

By Definition of $FJ - CI_\mu$, the total fuzzy preference value of $S_i \left(i = 1,2,\cdots,m\right)$ is:

$$FJ - CI_\mu\left(\tilde{\phi}_{i1}^c, \tilde{\phi}_{i2}^c, \cdots, \tilde{\phi}_{in}^c\right) = \lambda\left\{\oplus_{j=1}^n \tilde{\phi}_{ij}^c\zeta_{ij}\right\} \oplus \left(1-\lambda\right)\left\{\oplus_{j=1}^n \tilde{\phi}_{ij}^c\psi_{ij}\right\}. \tag{12}$$

Stage 4: Rank all alternatives.

According to Definition 1, all alternatives $S_i \left(i = 1,2,\cdots,m\right)$ are ranked. The bigger the $FJ - CI_\mu\left(\tilde{\phi}_{i1}^c, \tilde{\phi}_{i2}^c, \cdots, \tilde{\phi}_{in}^c\right)$, the better the $S_i \left(i = 1,2,\cdots,m\right)$.

5. CASE STUDY

With the increasing aging of the population, the continuous update of the concept of pension, the increasingly active pension market, the pension model tends to be diversified, and children are busy with work, there is no more time to take care of their parents, more and more elderly people choose to enjoy their old age in the nursing home. All kinds of nursing homes appear in everyone's eyes. However, how to choose a nursing home that really meets the needs and provides quality services, so that the elderly and their families can rest assured, this is a problem that many families are confused about.

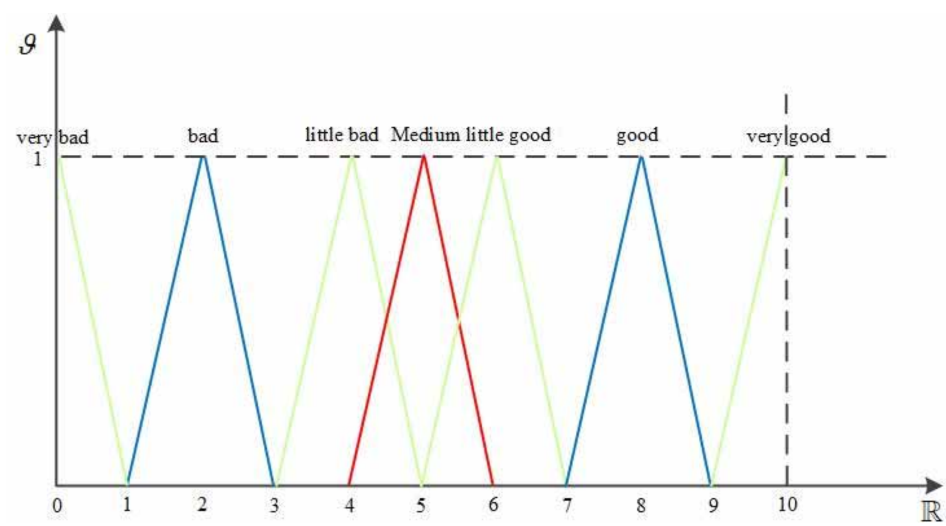
Mr. Chen, the children of a working family in Shanghai, is planning to choose a quality nursing home for their elderly parents. There are many nursing homes in Shanghai, but they do not know much about these nursing homes and do not know which one to choose. So, through his two consulting friends who know more about nursing homes, he is planning to choose from three qualified nursing homes $S = \{S_1, S_2, S_3\}$. And the evaluation criteria of nursing homes consider the following four criteria:

(1) The living environment of the nursing home (q_1);
(2) The basic facilities of the nursing home (q_2);
(3) The basic literacy of nursing staff in nursing homes (q_3);
(4) Medical conditions in nursing homes (q_4).

Moreover, Mr. Chen and his two friends form a team, denoted as $D = \{dm^1, dm^2, dm^3\}$, to give their respective fuzzy preference matrices $\tilde{F}^l = \left(\tilde{\phi}_{ij}^l\right)_{m\times n}$ $(l = 1,2,3.)$ about (S_i, q_j) $(i = 1,2,3; j = 1,2,3,4)$ by using the following fuzzy numbers: $\tilde{\phi}_0 = (0,0,1) \Leftrightarrow$ "*very bad*"; $\tilde{\phi}_1 = (1,2,3) \Leftrightarrow$ "*bad*"; $\tilde{\phi}_2 = (3,4,5) \Leftrightarrow$ "*little bad*"; $\tilde{\phi}_3 = (4,5,6) \Leftrightarrow$ "*Medium*"; $\tilde{\phi}_4 = (5,6,7) \Leftrightarrow$ "*little good*"; $\tilde{\phi}_5 = (7,8,9) \Leftrightarrow$ "*good*"; $\tilde{\phi}_6 = (9,10,10) \Leftrightarrow$ "*very good*", which the transformed images of linguistic and triangular fuzzy numbers are shown in Figure 4.

$$\tilde{F}^1 = \begin{pmatrix} (2,3,4) & (7,8,9) & (4,5,6) & (9,10,10) \\ (0,0,1) & (1,2,3) & (7,8,9) & (4,5,6) \\ (1,2,3) & (9,10,10) & (0,0,1) & (7,8,9) \end{pmatrix}, \tilde{F}^2 = \begin{pmatrix} (1,2,3) & (7,8,9) & (3,4,5) & (9,10,10) \\ (0,0,1) & (1,2,3) & (7,8,9) & (4,5,6) \\ (4,5,6) & (3,4,5) & (5,6,7) & (4,5,6) \end{pmatrix},$$
$$\tilde{F}^3 = \begin{pmatrix} (1,2,3) & (5,6,7) & (4,5,6) & (5,6,7) \\ (0,0,1) & (3,4,5) & (5,6,7) & (9,10,10) \\ (4,5,6) & (3,4,5) & (7,8,9) & (1,2,3) \end{pmatrix}.$$

Figure 4. The transformed images of linguistic and triangular fuzzy numbers



5.1. Calculation Steps

1) Obtain the group fuzzy preference matrix

According to Eq. (1), the fuzzy preference matrices of all the above decision makers are transformed into the following preference matrices with interval-valued functions:

$$\left[\tilde{F}^1\right]^\eta = \begin{pmatrix} [2+\eta, 4-\eta] & [7+\eta, 9-\eta] & [4+\eta, 6-\eta] & [9+\eta, 10] \\ [0, 1-\eta] & [1+\eta, 3-\eta] & [7+\eta, 9-\eta] & [4+\eta, 6-\eta] \\ [1+\eta, 3-\eta] & [9+\eta, 10] & [0, 1-\eta] & [7+\eta, 9-\eta] \end{pmatrix},$$

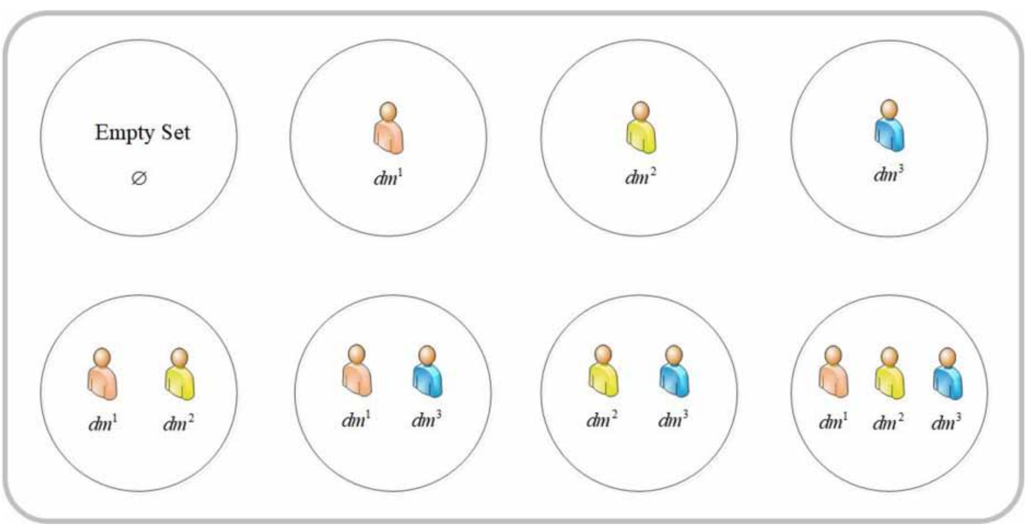
$$\left[\tilde{F}^2\right]^\eta = \begin{pmatrix} [1+\eta, 3-\eta] & [7+\eta, 9-\eta] & [3+\eta, 5-\eta] & [9+\eta, 10] \\ [0, 1-\eta] & [1+\eta, 3-\eta] & [7+\eta, 9-\eta] & [4+\eta, 6-\eta] \\ [4+\eta, 6-\eta] & [3+\eta, 5-\eta] & [5+\eta, 7-\eta] & [4+\eta, 6-\eta] \end{pmatrix},$$

$$\left[\tilde{F}^3\right]^\eta = \begin{pmatrix} [1+\eta, 3-\eta] & [5+\eta, 7-\eta] & [4+\eta, 6-\eta] & [5+\eta, 7-\eta] \\ [0, 1-\eta] & [3+\eta, 5-\eta] & [5+\eta, 7-\eta] & [9+\eta, 10] \\ [4+\eta, 6-\eta] & [3+\eta, 5-\eta] & [7+\eta, 9-\eta] & [1+\eta, 3-\eta] \end{pmatrix}.$$

Since the weights of $dm^l\ (l = 1,2,3)$ in this paper are calculated by the Shapley value based on fuzzy measure, it is a global interaction relationship between dm^1, dm^2 and dm^3. Hence, there are a total of $2^3 = 8$ coalitions of dm^1, dm^2 and dm^3. And the 8 cases for different coalitions of three decision makers $dm^l\ (l = 1,2,3)$ are mainly shown in Figure 5.

Let fuzzy measures of decision makers' coalitions be $\mu(dm^1) = 0.1, \mu(dm^2) = 0.25,$ $\mu(dm^1, dm^2) = 0.4, \mu(dm^1, dm^3) = 0.8, \mu(dm^2, dm^3) = 0.6, \mu(dm^1, dm^2, dm^3) = 1$ and $\mu(\varnothing) = 0$. By Eq.(8), it has $\chi_1 = 0.34, \chi_2 = 0.25$ and $\chi_3 = 0.41$. Then, the group preference matrix $\left[\tilde{F}^c\right]^\eta$ with interval-valued function is

Figure 5. The 8 cases for different coalitions of three decision makers $dm^l\ (l = 1,2,3)$



$$\left[\tilde{F}^c\right]^\eta = \begin{pmatrix} \left[1.34+\eta, 3.34-\eta\right] & \left[6.18+\eta, 8.18-\eta\right] & \left[3.75+\eta, 5.75-\eta\right] & \left[7.36+\eta, 8.77-0.41\eta\right] \\ \left[0, 1-\eta\right] & \left[1.82+\eta, 3.82-\eta\right] & \left[6.18+\eta, 8.18-\eta\right] & \left[6.05+\eta, 7.64-0.59\eta\right] \\ \left[2.98+\eta, 4.98-\eta\right] & \left[5.04+\eta, 6.7-0.66\eta\right] & \left[4.12+0.66\eta, 5.78-\eta\right] & \left[3.79+\eta, 5.79-\eta\right] \end{pmatrix}.$$

2) The best nursing home choice

Based on the Definition of $FJ-CI_\mu$ Hence, there are a total of $2^4 = 16$ coalitions of q_1, q_2, q_3 and q_4 of the nursing home. But, the q_1, q_2, q_3 and q_4 of the nursing home also belong to global interactions. Thus, the different coalitions of the four evaluated attributes $q_j\left(j=1,2,3,4\right)$ of the corresponding nursing home have 16 cases as shown in Figure 6.

Let the fuzzy measures of the attributes' coalitions of the nursing home be $\mu\left(\left\{q_1\right\}\right)=0.2\mu\left(\left\{q_2\right\}\right)=0.15, \mu\left(\left\{q_3\right\}\right)=0.3,$ $\mu\left(\left\{q_4\right\}\right) = 0.2.$ Since it has $\mu\left(\left\{q_1,q_2,q_3,q_4\right\}\right)=1$, by Eq. (3), it gets $\varepsilon = 0.527$. Moreover, according to Eq. (2), it gets

$$\mu\left(\left\{q_1,q_2\right\}\right)=0.3658, \mu\left(\left\{q_1,q_3\right\}\right)=0.5316, \mu\left(\left\{q_1,q_4\right\}\right)=0.4211, \mu\left(\left\{q_2,q_3\right\}\right)=0.4737,$$
$$\mu\left(\left\{q_2,q_4\right\}\right)=0.3658, \mu\left(\left\{q_3,q_4\right\}\right)=0.5316, \mu\left(\left\{q_1,q_2,q_3\right\}\right)=0.7236,$$
$$\mu\left(\left\{q_1,q_2,q_4\right\}\right)=0.6044, \mu\left(\left\{q_1,q_3,q_4\right\}\right)=0.7877, \mu\left(\left\{q_2,q_3,q_4\right\}\right)=0.7236.$$

By Definition 4, it has $\mu\left(\varnothing\right)=0$. By Definition of $FJ-CI_\mu$, for each $S_i\left(i=1,2,3\right)$, the following result is obtained regarding the ranking of preference values for each $q_j\left(j=1,2,3,4\right)$ of the nursing home.

- For S_1 , the preferences ranking about q_1, q_2, q_3 and q_4 of the nursing home is $\tilde{\phi}_{11}^c \preceq \tilde{\phi}_{13}^c \preceq \tilde{\phi}_{12}^c \preceq \tilde{\phi}_{14}^c$ for each $\eta \in \left[0,1\right]$.
- For S_2 , the preferences ranking about q_1, q_2, q_3 and q_4 of the nursing home is $\tilde{\phi}_{21}^c \preceq \tilde{\phi}_{22}^c \preceq \tilde{\phi}_{24}^c \preceq \tilde{\phi}_{23}^c$ for each $\eta \in \left[0,1\right]$.
- For S_3 , the preferences ranking about q_1, q_2, q_3 and q_4 of the nursing home is $\tilde{\phi}_{31}^c \preceq \tilde{\phi}_{34}^c \preceq \tilde{\phi}_{33}^c \preceq \tilde{\phi}_{32}^c$ for each $\eta \in \left[0,1\right]$.

And by Eq. (11), it has

$$\zeta = \begin{pmatrix} 0.2764 & 0.1658 & 0.3578 & 0.2 \\ 0.2764 & 0.192 & 0.3 & 0.2316 \\ 0.2764 & 0.15 & 0.3237 & 0.2499 \end{pmatrix} \tag{13}$$

and

$$\psi = \begin{pmatrix} 0.2 & 0.192 & 0.3316 & 0.2764 \\ 0.2 & 0.1658 & 0.3956 & 0.2386 \\ 0.2 & 0.2123 & 0.3666 & 0.2211 \end{pmatrix}. \tag{14}$$

Figure 6. The 16 cases of different coalitions for the $q_j\left(j=1,2,3,4\right)$ of the nursing home

By Definition of $F-CI_{\mu}$ and Eq. (13), it obtains

$$\left[F-CI_{\mu}\left(\tilde{\phi}_{11}^{c},\tilde{\phi}_{12}^{c},\tilde{\phi}_{13}^{c},\tilde{\phi}_{14}^{c}\right)\right]^{\eta}=\left[4.209+\eta,6.091-0.882\eta\right]$$
$$\left[F-CI_{\mu}\left(\tilde{\phi}_{21}^{c},\tilde{\phi}_{22}^{c},\tilde{\phi}_{23}^{c},\tilde{\phi}_{24}^{c}\right)\right]^{\eta}=\left[3.605+0.7236\eta,5.233-0.8944\eta\right]$$
$$\left[F-CI_{\mu}\left(\tilde{\phi}_{31}^{c},\tilde{\phi}_{32}^{c},\tilde{\phi}_{33}^{c},\tilde{\phi}_{34}^{c}\right)\right]^{\eta}=\left[3.860+0.8899\eta,5.699-0.949\eta\right].$$

Moreover, based on Definition of $FR-CI_{\mu}$ and Eq. (14), it gets

$$\left[FR-CI_{\mu}\left(\tilde{\phi}_{11}^{c},\tilde{\phi}_{12}^{c},\tilde{\phi}_{13}^{c},\tilde{\phi}_{14}^{c}\right)\right]^{\eta}=\left[4.732+\mu,6.569-0.8369\eta\right]$$
$$\left[FR-CI_{\mu}\left(\tilde{\phi}_{21}^{c},\tilde{\phi}_{22}^{c},\tilde{\phi}_{23}^{c},\tilde{\phi}_{24}^{c}\right)\right]^{\eta}=\left[4.190+0.8\eta,5.892-0.902\eta\right]$$
$$\left[FR-CI_{\mu}\left(\tilde{\phi}_{31}^{c},\tilde{\phi}_{32}^{c},\tilde{\phi}_{33}^{c},\tilde{\phi}_{34}^{c}\right)\right]^{\eta}=\left[4.014+0.875\eta,5.818-0.928\eta\right].$$

Next, let $\lambda=0.5$ and Definition of $FJ-CI_{\mu}$, it has

$$\left[FJ-CI_{\mu}\left(\tilde{\phi}_{11}^{c},\tilde{\phi}_{12}^{c},\tilde{\phi}_{13}^{c},\tilde{\phi}_{14}^{c}\right)\right]^{\eta}=\left[4.471+\eta,6.330-0.859\eta\right]$$
$$\left[FJ-CI_{\mu}\left(\tilde{\phi}_{21}^{c},\tilde{\phi}_{22}^{c},\tilde{\phi}_{23}^{c},\tilde{\phi}_{24}^{c}\right)\right]^{\eta}=\left[3.897+0.7618\eta,5.563-0.8982\eta\right]$$
$$\left[FJ-CI_{\mu}\left(\tilde{\phi}_{31}^{c},\tilde{\phi}_{32}^{c},\tilde{\phi}_{33}^{c},\tilde{\phi}_{34}^{c}\right)\right]^{\eta}=\left[3.937+0.8824\eta,5.758-0.9385\eta\right].$$

For each $\eta\in\left[0,1\right]$, by Definition 1, it has

$$FJ-CI_{\mu}\left(\tilde{\phi}_{21}^{c},\tilde{\phi}_{22}^{c},\tilde{\phi}_{23}^{c},\tilde{\phi}_{24}^{c}\right)\preceq FJ-CI_{\mu}\left(\tilde{\phi}_{31}^{c},\tilde{\phi}_{32}^{c},\tilde{\phi}_{33}^{c},\tilde{\phi}_{34}^{c}\right)\preceq FJ-CI_{\mu}\left(\tilde{\phi}_{11}^{c},\tilde{\phi}_{12}^{c},\tilde{\phi}_{13}^{c},\tilde{\phi}_{14}^{c}\right).$$

Thus, S_1 is chosen as the best nursing home.

In summary, based on the advantage of interval-valued functions, the above computational results are ranked the same under each η-cut set with $\eta\in\left[0,1\right]$. This shows the reliability and rationality of the $FJ-CI_{\mu}$ based on interval-valued functions.

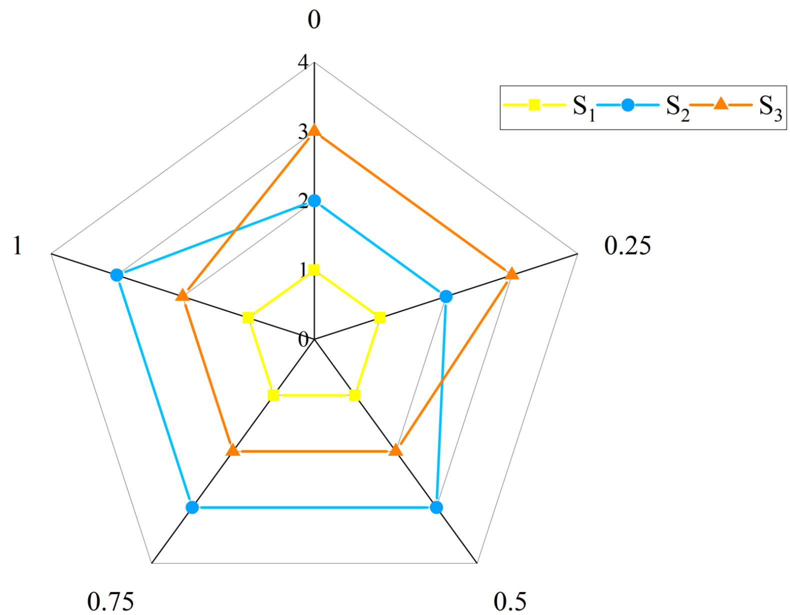
5.2. Sensitivity Analysis

To better explain the advantages of the proposed technique, it performs the following sensitivity analysis. The ranking results of nursing homes S_1, S_2 and S_3 with potential under different parameters λ are shown in Table 1 and Figure 7 below.

Table 1. Ranking of S_1, S_2 and S_3 under different parameters λ

λ	Ranking
0	$S_1 > S_2 > S_3$
0.25	$S_1 > S_2 > S_3$
0.5	$S_1 > S_3 > S_2$
0.75	$S_1 > S_3 > S_2$
1	$S_1 > S_3 > S_2$

Figure 7. Ranking of nursing homes S_1, S_2 and S_3 with qualifications under different parameters λ



From Table 1 and Figure 7, it can be obtained that the most optimal nursing home S_1 is constant, which clarifies the reliability of the proposed method. However, as the parameter λ changes, the ranking results of S_2 and S_3 are transformed when the parameter λ is varied from 0 to 1. In particular, when the parameter λ is equal to 1, $FJ-CI_\mu$ is degenerated to $F-CI_\mu$. The corresponding ranking results of alternative nursing home under the $F-CI_\mu$ is $S_1 > S_3 > S_2$. When the parameter λ is equal to 0, $FJ-CI_\mu$ is degenerated to $FR-CI_\mu$. The corresponding ranking results of alternative nursing home under the $FR-CI_\mu$ is $S_1 > S_2 > S_3$. It can be seen that the ranking results of $F-CI_\mu$ and $FR-CI_\mu$ are different. In summary, this not only shows the advantage of $FJ-CI_\mu$ but also clarifies the flexibility of the proposed technique.

5.3. Comparative Analysis

In order to better show the effectiveness and reliability of this paper, this paper makes the following comparative analysis with the existing references, as shown in Table 2.

Table 2. Comparative analysis with the existing references

References	Preference Expression	Whether to Retain the Original Opinion	Relationships Among Decision Makers	Attributes Interaction
Wang et al. (2018)	Fuzzy number	No	No	Local interaction
Teng et al. (2021)	Probabilistic linguistic	No	No	Local interaction
Meng et al. (2021)	Exact number	Yes	No	Global interaction
The proposed Method	Fuzzy number	Yes	Yes	Global interaction

Although fuzzy numbers (Wang et al. 2018) or probabilistic language (Teng et al. 2021) are used to express their opinions, they do not retain the original opinions of experts in the process of opinion aggregation, and decision makers do not communicate with each other. These proposed methods of Choquet integral integration are merely local interactions between attributes.

In order to deal with the problem of local interaction between attributes, Meng et al. (2021) proposed bidirectional Choquet integral. However, they only use exact numbers to express their opinions, and do not reflect the interaction between decision makers. The interval-valued function can not only express their own opinions well, but also guarantee the original opinions of decision makers. Inspired by these, the fuzzy joint Choquet integral based MAGDM under interval-valued function proposed in this paper not only reflects the interactive relationship between decision makers to retain the original opinions of decision makers, but also reflects the global interaction between attributes. Therefore, the method proposed in this paper has certain advantages compared with the traditional references.

6. CONCLUSION AND FUTURE WORKS

A new $FJ-CI_{\mu}$ based MAGDM approach under interval-valued function is put forward and then this proposed method is applied to the selection of nursing homes, which it mainly includes the following three contributions:

(1) The $FJ-CI_{\mu}$ based on an interval-valued function is proposed. First, the fuzzy numbers with interval-valued functions are applied through decision makers to depict their preferences in complex and uncertain environments. And then, the $F-CI_{\mu}$ and the $FR-CI_{\mu}$ based on interval-valued functions are put forward, respectively. And the $FJ-CI_{\mu}$ based on an interval-valued function is designed, which is composed of the linear convex combination of the $F-CI_{\mu}$ and the $FR-CI_{\mu}$ with interval-valued functions. This not only reflects the interaction between multiple evaluation attributes, but also retains the initial preference of the decision maker.
(2) The new fuzzy MAGDM approach is presented. First, the Shapley value with FM*s* is applied to assign the each decision maker's weight, which illustrates the global interaction relationships among decision makers to ensure fairness of decision makers. Moreover, the fuzzy group preference matrix is acquired through fusing the fuzzy preference matrices of all decision makers.
(3) The proposed technique is applied to the nursing home selection by family members, and the final results not only reflect the global interaction between members, but also clarify that there is a certain positive correlation between different evaluation attributes of nursing homes. Therefore, the proposed technique is effective and reliable.

This article also has some limitations. Since decision makers have different knowledge levels, ages and identities, these decision makers have certain differences. Therefore, in the future, the technology proposed in this paper will be integrated into the group consensus model (Liu et al.,2019; Lu et al.,2021; Cao et al.,2021) to explore the impact of multiple attributes interaction on consensus results. In addition, the techniques presented in this paper can also be applied to other fields (Yu, 2024; Tang, 2023, Wang & Chen, 2023; Guo & Zheng,2023). such as network security (Yu, 2024).

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